On The Undergraduate Program For Majors In Mathematics

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The progress of mathematics and mathematics education in the Philippines depends greatly on the quality of the curriculum for the undergraduate majors. Cognizant of this, the Mathematical Society of the Philippines in 1973 recommended a strong undergraduate program in mathematics. A survey of the present mathematics situation in colleges of the country undertaken by the Society reveals that the programs in mathematics in many institutions are sadly lacking in the basic requirements.

The objective of this paper is to create a desirable impact, deep and broad on the undergraduate mathematics programs in colleges and universities in the country. Acting as members of the Mathematical Society of the Philippines, Dr. Jose Marasigan of the Ateneo de Manila University, Prof. Imelda Diaz of De La Salle University, and I formed a committee for the undergraduate mathematics program and held a number of meetings to formulate a nucleus of an appropriate curriculum for undergraduate majors in mathematics for Philippine institutions of higher learning. This committee will soon put up a project where concerned faculty will share their mathematical and teaching expertise by giving special lectures on course contents and curriculum development to college mathematics teachers who desire to enrich their capabilities. The recommendations in this paper were presented in a seminar sponsored by the Society and the Mathematics Department of the Far Eastern University in February of this year. It was the consensus of the participants in this seminar that mathematics departments should prepare their faculty members to adopt the recommendations in the near future.

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This section is a report on the curricular changes which the undergraduate program in mathematics has undergone in the institution. I am most familiar with the University of the Philippines at the Diliman Campus. Let me go back in the near past of about thirty years ago. Analytic geometry and calculus were taught as separate courses and the concept of function and limit had not yet earned the limelight they enjoy today. Beyond the calculus, the mathematics major studied

1. Advanced plane geometry
2. Projective geometry
3. Theory of equations (abolished in the late fifties)
4. Differential equations
5. Fundamental concepts of algebra and geometry (in the flavor of Young's book of the same title and Veblen's Foundation of Geometry)
6. Eight units of physics, mechanics, sound and electricity.

Advanced calculus, abstract algebra and complex variables were taken up in graduate school and it was only in the fifties that these courses were required of the undergraduates. In the later part of the fifties, formal courses in

1. Number theory
2. Set theory
3. Statistics
4. Linear algebra
5. Vector analysis
6. Numerical analysis

were instituted to form a set of courses from which electives could be chosen. With the advent of the sixties, courses in

1. Finite differences
2. Boolean algebra
3. Actuarial science
4. Finite mathematics
5. Probability theory
6. Real analysis
7. Elementary topology

for the undergraduate were instituted. The present decade witnesses the addition of

1. Mathematics for decision making
2. Mathematics for the social and behavioral sciences

3. Mathematical methods of operations research
4. Linear programming
5. Introduction to computer programming

which as can be observed from the titles are the department's answer to the basic needs of applied mathematics.

The shift to emphasis in conceptual thinking began in 1955 at the same time that analytic geometry and calculus were taught as an integrated course, not only to the mathematics major but also to all students in the University of the Philippines whose degree courses require calculus. A few years later, vector algebra started to be used in the teaching of analytic geometry and calculus. Such is a brief account of the growth of the undergraduate mathematics offerings and the changes in the B.S. mathematics curriculum in the University.

What factors brought about these changes? Interaction with other units and departments of the University brings to the attention of the mathematics department the mathematics needs of their students. Since the Mathematics Department of the University of the Philippines in the Diliman Campus serves all the units in the courses labeled as mathematics, it is just reasonable that the department should meet their needs as closely as possible. Advancing frontiers of knowledge, changes in technology, the coming of the computer, better rewards in applied fields, the new mathematical approaches and the new expertise brought home by returning faculty members from their studies abroad are some of the factors which bring about the continuing reexamination of the mathematics curriculum in the University of the Philippines.

More than a quarter of a century ago, the education of the mathematics major in our country was towards his becoming a teacher of mathematics or preparing him for graduate school. Today, the mathematics major graduate can choose a career in industry, management consultancies, economic policy-making bodies, government service, etc. Since a wide choice of a career is open to the qualified mathematics major, an appropriate preparation must be provided.

Mathematics has been growing at such a phenomenal rate within the discipline itself and its relation to other sciences, that the selection of what mathematics courses should be included in a B.S. Mathematics curriculum is of vital issue. It is now the conviction that career choices should be more tentative and reversible than it was before. This belief is widespread among our students and even among our faculty members. This is shown by the fact that faculty members, especially among the younger group in our department are

showing interest in the new fields that mathematics serves, like actuarial science, programming and computer science and some even traverse departmental boundaries to go to management and economics or at least expand their own intellectual area. For this reason, it is my opinion that we increase the amount of choice which the students can have in making a program of study, provided a set of required core courses is fixed. It is now the thinking of the students and alumni that the curriculum should not only meet the needs of mathematics in academe but surely must also meet the needs of industry and government. Because of these needs, we must clearly define the goals of our undergraduate mathematics program.

What are the goals of the undergraduate program in mathematics and how well does the proposed curriculum meet such goals? Is it to prepare the student to teach? Is it to train him for graduate work? Is it to prepare him for the needs of industry? Although student career options are sensitive to employment expectations, still one's decision is guided by one's sense of values, interest, and advancement and not by mere economic reason. There ought to be options available for those who will go to teaching, for those who will go into research, for those who will go to industry or government, but, I believe, too much specialization should not be given to the undergraduate. There is a growing mathematization of the biological, behavioral and policy sciences, hence experts in operations research are in demand. Many of the new applications in mathematics require background in computer science, programming, graph theory and statistics, hence we must make these courses within the reach of the undergraduates.

A curriculum limited to these utilitarian objectives is a very restricted one; there must be broader goals towards teaching mathematics in its own right both in its internal structure and its relation to other disciplines. For the academic and applied mathematician, conceptual accuracy, clarity and precision of statements are of prime concern, for it is the ability to abstract and formulate the problem briefly and clearly which makes the mathematician indispensable. It must be the final aim of the program to develop in the student mathematical taste of high quality, intellectual honesty and a strong commitment to his own self-improvement.

We would like to focus our attention on a central curriculum beginning with college algebra, trigonometry and calculus and recommend a curriculum which will serve the goals of the undergraduate mathematics program and the needs of the jobs which the mathematics major seeks to enter.

The recommendations of our committee follow:

Required courses

College Algebra and Trigonometry	6 units (9*)
Analytic Geometry and Calculus	13 or 15 units
Probability and Statistics	6 units
Differential Equations	3 units
Linear Algebra	3 units
Abstract Algebra	3 units
Advanced Calculus	3 units
Complex Analysis	3 units
Modern Geometry	3 units

Electives

Real Analysis	Probability Theory
Elementary Topology	Combinatorics
Set Theory	Methods of operations Research
Number Theory	Introduction to Computer Programming
Graph Theory	Linear Programming
Vector Analysis	Life Contingencies
Finite Differences	Mathematics of Decision Making
Actuarial Science	

*An additional non-credit course in algebra is needed for students with a weak background.

We maintain that there should be a strong core of pure mathematics, namely algebra, analysis and geometry and even as we recognize the necessity of new areas to be developed, this core is still the basic ingredient of the younger areas of mathematics because in these areas the same language and the same spirit of mathematics prevail.

For a variety of reasons, in this present age of computers and management-oriented courses, there need not be just a single curriculum for mathematics majors. These recommendations are not intended to be prescriptive but flexible enough so that colleges can plan within their resources a maximally productive curriculum for their students. These recommendations also aim to urge colleges to bring their offerings to the standard which the Mathematical Society of the Philippines recommends. Our committee therefore, presents

the descriptions of and commentaries on the above required courses which attempt to convey some ideas on the manner and vitality that we feel appropriate in teaching these courses.

It is common knowledge that college algebra, trigonometry and calculus with analytic geometry are basic mathematics courses for the undergraduate mathematics majors and therefore the rationale of their inclusion in the curriculum is clear. It is hoped however, that we can relegate the teaching of algebra and trigonometry to the high school within the next five years, so that college mathematics would begin with analytic geometry and calculus. The teaching of trigonometry should emphasize circular functions of real numbers rather than confining it only to the solution of right triangles. It has been noted that there are still many mathematics departments that offer analytic geometry and calculus as separate courses. The experience that we have in our universities is that an integrated course gives the students greater assimilation and power of transfer. The unifying aspect of an integrated course is clearly significant because the student realizes that the subjects are not separated but are closely interrelated. It is easier to absorb and remember related topics whose materials are developed through consistent application of a few general methods of procedure.

As much as possible, elementary calculus should be taught intuitively, in the sense that whenever a new concept is to be introduced, much motivation must be done first via examples familiar to the students. It is important that theorems and definitions be stated with care and examples and counterexamples should be shown supporting the need of the hypotheses. Whenever the student is prepared to appreciate a proof, it should be presented. Calculus deals with concepts of limits, continuity, derivative, integral and convergence and the teacher must realize that the concrete examples of these concepts should facilitate the student's understanding of their abstractness.

Some of the most abstract mathematical systems are in abstract algebra. The undergraduate training of the mathematics major today is incomplete without this. The ability of the mathematician to abstract is developed in abstract algebra where mathematical structures are constructed by collecting a set of elements and formulating an arbitrary set of rules for them which need only to be consistent.

Linear algebra is now an important tool of the engineer, physicist, economist and even the social scientists so that the mathematics major is certainly required to know more of it. The notions of vector space and linear transformation which are basic in linear algebra clarifies the students concept of space.

Differential equations are an indispensable tool in physics and other applied fields. The undergraduate should be familiar with ordinary differential equations. He must be skilled in the use of operators and Laplace transforms in the solution of linear differential equations. Solutions by series and numerical methods should not be left out.

Advanced calculus is the key to graduate work in analysis. This does not mean that if the undergraduate major has no ambition to graduate work, he should be exempted from this. The student should be able to develop mathematical finesse and maturity in this course. Many concepts, algebraic, geometric, topological, and analytic are discussed here, so that a well-rounded mathematics students will be its product if well transmitted and received. Although a geometrical viewpoint is of much help in the study of functions of one, two, or more variables, a considerable advance in conceptual depth can be made here. Topological concepts painstakingly and clearly explained with many concrete examples will facilitate a better understanding of proofs of theorems. The concept of area must precede the theory of integration. The effect of transformations on derivatives and integrals from n-space to m-space should be given much discussion. Functional dependence and implicit function theorems must be related to the Jacobian. Some minor independent study can be required in a six-unit course.

The theory of functions of a complex variable is prominent in pure and applied mathematics because many concepts become clarified and unified when expressed in complex variable theory. Being an analysis course, the concepts of limits, convergence, derivatives and integration are studied. Analytic functions and their properties constitute a great part of the course. The residue theorem which in itself is beautiful becomes a powerful tool in evaluating some real integrals which prove difficult to evaluate in real variables.

Statistics and probability are now standard tools of research in life and social sciences and are prerequisites to applied mathematics. The mathematics major should be in a better position than others to use these as tools.

Geometry should be a part of the undergraudate Mathematics program because it is one of the traditional branches of mathematics and because it exerts a strong influence in the development of mathematics. The student should be familiar not only with Euclidean geometry but also non-Euclidean, affine and projective geometries.

For the future of mathematics education in our country let us make it our crucial responsibility as college mathematics teachers, as chairmen of math departments and as members of the MSP to participate jointly and severally in questions of curriculum reform

and engage in conscious planning for the improvement of the quality of teachers and the teaching of mathematics.

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